Einstein’s Terrain

Dewi Becu
Reyhan Danisan
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The study

This paper will discuss the aperiodic Einstein tile, nicknamed the "hat polykite", where this novel design defies traditional tiling patterns by avoiding recurrence. The research done for this project showed us that the Einstein tile is more than just a geometric shape; it is an opportunity to reconsider our views on symmetry and periodicity in tiling. In this study, we aim to present the hat polykite by investigating how its unique shape enriches our understanding of tiling patterns.

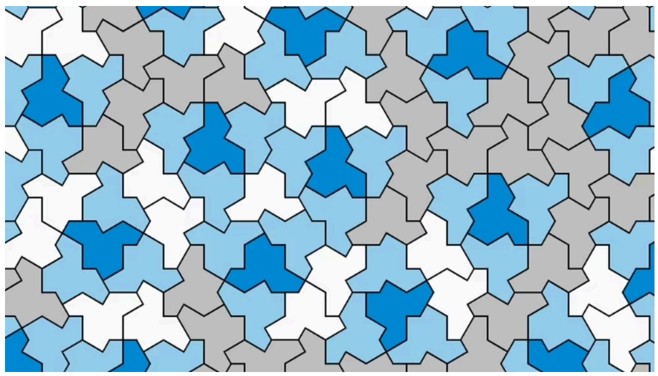

Figure 1: Einstein tile put together to form a non-repeating pattern

The hat polykite has emerged as a peculiarity in the world of tiling. This shape is an example of aperiodicity, as it refuses to comply with repeating patterns, hence the name "einstein," which signifies a single shape that tiles aperiodically. To prove this distinct feature, the study uses a dual-method approach. The first method involves an analytical investigation, systematically ruling out the likelihood of the polykite producing regular, repeating patterns. This analytical process is similar to solving a complex puzzle with a constantly changing layout, where each attempt to find a repeating sequence uncovers new layers of complexity. The second method examines the polykite in further detail by dividing the tile into smaller triangular portions that follow specific rules, see Figure 2. This analysis of the smaller segments not only supports the polykite's aperiodic qualities but also reveals subtle variations in its tiling behaviour, showing the intricate connection between individual parts and the overall pattern. It emphasizes how the hat polykite defies typical geometric tiling norms.

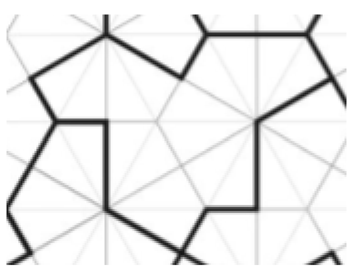

Figure 2: Triangular segment within the Einstein tile

These smaller segments within the Einstein tile consist of a multitude of small kite-like shapes that are repeated multiple times, flipped or unflipped, in order to create the final “hat” polykite shape. After forming the tile, it is possible to create multiple clusters that will form the overall aperiodic tiling. There are many ways to go about the placement of the tiles, each leading to a unique arrangement of the tiles. The layout of the tiles is guided by rules specifying how tiles can be next to one another. These rules ensure that despite the variety of placement options, the final pattern does not repeat frequently, preserving its aperiodic nature.

The study then broadens its scope to demonstrate how the hat polykite fits into the greater world of aperiodic tiling, in which shapes do not form repeating patterns. It introduces a substitution technique, which explains how the polykite may generate an infinite number of tiling designs, each unique from the others. This section of the study shows how combining rigid mathematical procedures with creative thinking can lead to fascinating discoveries. The incorporation of the golden ratio in these patterns makes them even more complex, suggesting a hidden, deeper order in their arrangement. The study also addresses 'fault lines' in these patterns, a novel concept that allows us to better grasp the random yet structured aspect of aperiodic tiling.

Our research of the hat polykite has caused us to reconsider assumptions in the field of geometric tiling. This study not only revealed the intricate nature of this unusual form but also paved the way for further exploration in the field of aperiodic tiling. It raises intriguing questions about how complex geometric patterns can be and opens up possibilities to find other shapes with similar aperiodic properties. This study shows how the field of mathematics is continually evolving, with each new discovery serving as an opportunity for future research and intellectual growth.

The Output

The product: *'Einstein's Terrain'*, is a 30 by 20 rectangular piece with over a hundred smaller wooden Einstein tiles laser cut and put together. They were put together to fit each other perfectly on the first layer, and multiple other tiles were glued on top of one another to have elevation in the overall piece. It outputted a wooden structure that resembles a terrain.

Figure 3: The output, Einstein's Terrain

With the discovery of this aperiodic tile, multiple uses can come out of it. The most straightforward one was to have it as a puzzle, as they are tiles that can be put together, akin to puzzles. However, taking this a step further shows further intersecting applications. For example, the textile industry has been taking off with 3d printing textiles, as explained in Spahiu's study (Spahiu et al., 2020). These printed fabrics use geometric shapes to construct the same cloth-feel. The discovery of the new aperiodic Einstein tile could launch a new manner of constructing textiles. Nevertheless, this tangent shows how this new tile could have many uses.

A similarity from how it appears akin to a puzzle is how it seemed similar to board games. Many board games use tiles to function, although most use symmetrical shapes like squares or hexagons and rarely use a bizarre-looking polygon such as our 'hat'-tile. This brought the following consideration of how the clustered Enstein tiles could represent a map. Taking regions in Europe as an example, countries' borders often follow geographical trends, such as borders being at mountains and often not straight: they are dented and curved borders. This is where a typical symmetrical and repetitive tile pattern may fall short. However, considering the Einstein tile is specifically made to be non-repetitive, lines following the shape's rip can be guided in a specific direction and could be a uniquely interesting tile representing map regions. See Figure 4 for a demonstration of this instance.

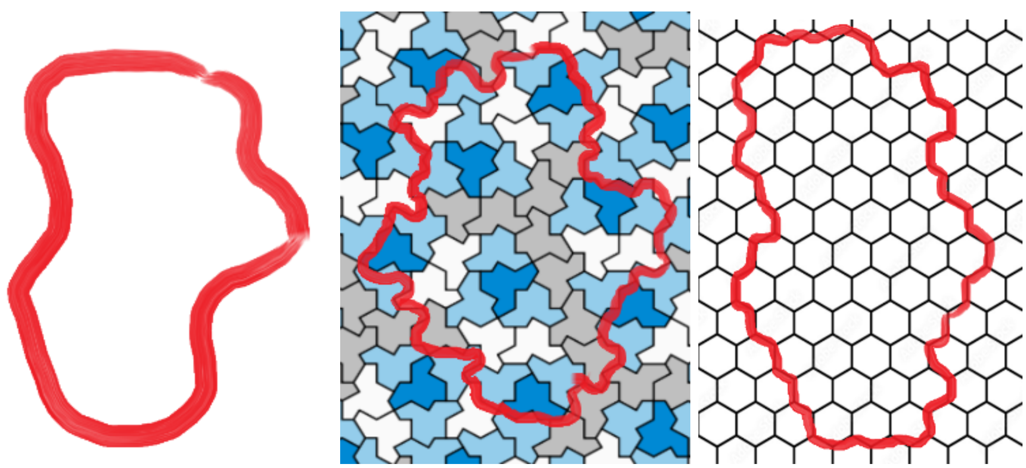

Figure 4: Comparison how the Einstein aperiodic tile (middle) and the hexagon tile (right) are constructed to be as similar to a fluid shape (left)

Therefore, *Einstein's Terrain* was created to show at a miniature scale how a terrain could be reconstructed using the Einstein tiles. The elevation is shown by having clusters of mountain ranges or with a lack of tiles, therefore showing what may resemble valleys. The team's observation shows that the non-repeating pattern seems to create a more organic atmosphere than a more structured grid.

While it is true that symmetric tiles, or tiles with a repeating pattern, have the advantage of allowing the user, or builder of the terrain, to manipulate the terrain more easily, the terrain cannot consistently be replicated in these structure's grids. One may argue that taking a hexagon as a shape would allow manipulation of all sides and that any terrain can be drawn out when a shape is small enough. Moreover, while true, the Einstein tile allows less structural freedom to simulate terrain. However, it is acknowledged that simulating cities or structured, grid-esque regions could be where the Einstien-tile lacks.

Lastly, the *Einstein's Terrain* is a physical rendition of a terrain simulation. Seeing how this holds up in a virtual simulation would prove interesting. This would allow further investigation if using the Einstien tile would be helpful in simulating terrain, such as recreating country borders, and see if the same-sized Enstien tile would perform as well as another shape in terms of terrain resemblance and performance.

References

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